18.06 (Fall '12) Problem Set 3

This problem set is due Thursday, September 27, 2012 by 4pm in 2-255. The problems are out of the 4th edition of the textbook. For computational problems as usual, please include a printout of the code with the problem set (for MATLAB in particular, `diary(“filename”)` will start a transcript session, `diary off` will end one, also copy and paste usually works as well.) .. or if more appropriate include a figure.

1. Do Problem 5 from 3.2.

2. Reduce these matrices to their ordinary echelon forms U:

   (a)
   $$\begin{pmatrix} 1 & 4 & 1 & 2 & 3 \\ 1 & 4 & -1 & 5 & 2 \\ 0 & 0 & -4 & 6 & 3 \end{pmatrix}$$

   (b)
   $$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 10 \\ 3 & 6 & 13 \end{pmatrix}$$

   Which are the free columns and which are the pivot columns?

3. Do Problem 25 from 3.2.

4. Do Problem 2 from 3.3.

5. Do Problem 12 from 3.3.

6. Do Problem 34 from 3.4.

7. Do Problem 21 from 3.5.

8. Do Problem 24 from 3.5.

9. *You do not need to touch MATLAB, or even the computer, for this problem.* Here is what happens when one uses MATLAB 's rank command:

   ```
   >> e=1e-15; a=[1+e 1;1  1]; rank(a)
   ans =
        1
   ```

   (the first command just means that $e = 10^{-15}$) Show that the rank is not mathematically correct. Why do you think MATLAB produces this answer? (No need to read MATLAB documentations - a couple of sentences with a reasonable guess would suffice). Is there a nearby matrix that really is rank 1?

10. This problem shows how linear algebra can compute interpolations of functions whose accuracy can far exceed the series expansions you learned in calculus. Recall the quartic approximation $e^x \approx 1 + x + x^2/2 + x^3/6 + x^4/24$. The file is available on the class web page, one can copy and paste the pdf, or just type yourself line at a time.

    We hope this code is not too hard for the non Matlab users to translate. We can give hints for other languages especially if you start early. If some of you translate this to other languages quickly, perhaps you can share with the rest of the class.

    The code below compares interpolation in blue with the series in green on the interval [1,2]. Plotted is the error on a log scale. Which is better?

    What we really want you to think about and explain is the equation A*(ci)=b; If ci are the coefficients of a quartic, argue that A*ci evaluates the quartic at the interpolation points. Then argue that solving A*ci=b amounts to finding the quartic that interpolate exp(x) at the interpolation points given in p.

```
%This program compares two  quartic approximations
%to y=exp(x) on the interval [1,2]

x=(1:.01:2)';
y=exp(x);    % the true answer

ct=[1/24 1/6 1/2 1 1];  % The  series, highest degree first
yt=polyval(ct,x);


p=linspace(1,2,5)';    % Five equal interpolation ponts


A=[p.^4 p.^3 p.^2 p.^1 p.^0];    % The interpolation matrix
b=exp(p);                       % The true values at the interpolation points
ci=A\b;                          % The coefficients of the quartic

yi=polyval(ci,x);                % Evaluate the interpolant
semilogy(x,abs([y-yi y-yt]));  % Compare on log scale interpolant error
                                 % (blue) vs taylor (green)
```

18.06 Wisdom: For many applications, the most important thing about a matrix is whether it is full rank or not.